

Symmetric division deg index of tricyclic and tetracyclic graphs

¹V. Lokesha and ²T. Deepika

^{1,2}Department of Mathematics, Vijayanagara Sri Krishnadevaraya University, Ballari

E-mail: v.lokesha@gmail.com, sastry.deepi@gmail.com

Abstract— The symmetric division deg index (SDD) is one of the 148 discrete Adriatic indices analyzed by Vukčević and Gašperov on the benchmark datasets of the International Academy of Mathematical Chemistry. SDD is a significant predictor of total surface area for polychlorobiphenyls. In this article, we characterize the SDD index for the class of all n -vertex tricyclic and tetra-cyclic graphs.

Index Terms — Symmetric division deg index, cyclomatic number, tricyclic graph, tetracyclic graph, pendant vertices, maximum degree and minimum degree.

1 INTRODUCTION

Let Σ denotes the class of all graphs, then a function $T : \Sigma \rightarrow R^+$ is known as topological index if for every graph H isomorphic to G , $T(G) = T(H)$.

Different topological indices are found to be useful in isomer discrimination, Quantitative structure-activity relationship (QSAR), Quantitative structure-property relationship (QSPR), pharmaceutical drug design, etc. in chemistry, biochemistry, medicine and nanotechnology [6-17].

If G has order n and size m containing k components, then $c = m - n + k$ is called the **cyclomatic number** of G then G is called unicyclic, bicyclic, tricyclic and tetracyclic, respectively.

In [1-4] some of the present authors computed the first and second maximum values of the Symmetric division deg index in the class of all n -vertex unicyclic and bicyclic graphs. Also computed first, second and third maximum values of atom-bond connectivity index in the class of all n -vertex tricyclic and tetracyclic graphs.

This paper is motivated from the works of C. K. Gupta, V. Lokesha et. al [3] we established the first and second maximum values of SDD index in all class of all n -vertex tricyclic(TrG) graphs [2]. The SDD index is defined as

$$SDD(G) = \sum_{uv \in E(G)} \frac{\max(d_u, d_v)}{\min(d_u, d_v)} + \frac{\min(d_u, d_v)}{\max(d_u, d_v)}$$

Many topological indices are bond-additive [5], they can be presented as a sum of edge contributions and have the following term:

$$\sum_{uv \in E(G)} f[h(u), h(v)]$$

Where $(x, t) \geq 1 \in Z^+$ and consider the function,

$$f(a, x) = \phi(x, a) - \phi(x, a - 1), a \geq 2.$$

Lemma 1: Let $G = TrG_{n,p}$ be the tricyclic graph of order n and size m with exactly p pendant vertices then

$$SDD(G) \leq \frac{p(\varepsilon_2 - 1)(\varepsilon_1^2 - \varepsilon_2) + n(\varepsilon_1^2 + \varepsilon_2^2)}{\varepsilon_1 \cdot \varepsilon_2}$$

where ε_1 and ε_2 are maximum and minimum degree.

Proof: Since G is a tricyclic graph with size $m = n + 2$, $4 \leq \varepsilon_1 \leq p + 3$. Thus

$$\begin{aligned} SDD(G) &= p \left(\frac{(p+3)^2 + 1}{p+3} \right) + (m-p) \left(\frac{(p+3)^2 + 9}{3(p+3)} \right) \\ &\leq p \left(\frac{p^3 + 6p + 10}{p+3} \right) + (n-p) \left(\frac{p^3 + 6p + 18}{3(p+3)} \right) \\ &\leq \frac{p(\varepsilon_2 - 1)(\varepsilon_1^2 - \varepsilon_2) + n(\varepsilon_1^2 + \varepsilon_2^2)}{\varepsilon_1 \cdot \varepsilon_2}. \end{aligned}$$

Hence the proof.

Lemma 2: The tricyclic graph $TrG_{n,p}$ where $n \in Z^+$, $0 \leq p \leq n - 4$ holds that

$$\begin{aligned} SDD(TrG_{n,p}) &> SDD(TrG_{n,p-1}) > SDD(TrG_{n,p-2}) > \\ SDD(TrG_{n,p-3}) &> \dots \dots \dots > SDD(TrG_{n,0}). \end{aligned}$$

Proof: Consider the function,

$$\alpha(p) = p \left(\frac{(p+3)^2 + 1}{p+3} \right) + (n-p) \left(\frac{(p+3)^2 + 9}{3(p+3)} \right)$$

Therefore,

$$\alpha'(p) = p \left(\frac{2p - p^2 + 8}{(p+3)^2} \right) + (n-p) \left(\frac{2p - p^2}{9(p+3)^2} \right)$$

$$+ \left(\frac{2(p^2 + 6p + 6)}{3(p + 3)} \right) > 0$$

Hence $\alpha(p)$ is increasing function for $0 \leq p \leq n - 4$ and

$$\alpha(1) = \frac{17}{4} + (n-1)\frac{25}{12} > \alpha(0) = 2n$$

Thus we have

$$\begin{aligned} SDD(TrG_{n,p}) &> SDD(TrG_{n,p-1}) > SDD(TrG_{n,p-2}) > \\ SDD(TrG_{n,p-3}) &> \dots \dots \dots > SDD(TrG_{n,0}) \end{aligned}$$

2 MAXIMUM VALUES OF SDD INDEX WITH FOUR NON-PENDANT VERTICES

Let n -vertex tricyclic graphs, $n \geq 4$ with four non-pendant vertices will be determined. We assume that $K_{4,p}(p_1, p_2, p_3, p_4)$ is a tricyclic graph obtained from the complete graph K_4 by attaching p_i vertices to v_i where $v_1, v_2, v_3, \dots, v_r$ be the vertices of c_r are consecutively labeled, and $p_i \geq 0$ for $i = 1, 2, \dots, r$ and $p_1 \geq p_2 \geq \dots \geq p_r$. Since $\sum_{i=1}^r p_i = n - r$.

Theorem 2.1: Let $G = K_{4,p}(p_1 - 1, p_2 - 2, p_3, p_4)$ where

$p_1 \geq p_2 \geq 2$ and $G' = K_{4,p}(p_1, p_2 - 1, p_3, p_4)$ then

$SDD(G) < SDD(G')$.

Proof: Consider

$$\begin{aligned} SDD(G') - SDD(G) &= [p_1\phi(1, p_1 + 3) - (p_1 - 1)\phi(1, p_1 + 2)] + \\ &\quad [(p_2 - 2)\phi(1, p_2 + 1) - (p_2 - 1)\phi(1, p_2 + 2)] + \\ &\quad [\phi(p_1 + 3, p_2 + 1) - \phi(p_1 + 2, p_2 + 2)] + \\ &\quad [\phi(p_1 + 3, p_3 + 2) - \phi(p_1 + 2, p_3 + 2)] + \\ &\quad [\phi(p_1 + 3, p_4 + 2) - \phi(p_1 + 2, p_4 + 2)] + \\ &\quad [\phi(p_2 + 1, p_4 + 2) - \phi(p_2 + 2, p_4 + 2)] + \\ &\quad [\phi(p_2 + 1, p_3 + 2) - \phi(p_2 + 2, p_3 + 2)] \\ &= \alpha(p_1) - \alpha(p_2 - 1) + f(p_3 + 2, p_1 + 3) + \\ &\quad f(p_4 + 2, p_1 + 3) - f(p_4 + 2, p_2 + 2) - \\ &\quad f(p_3 + 2, p_2 + 2) + \frac{(p_1 + 3)^2 + (p_2 + 1)^2}{(p_1 + 3)(p_2 + 1)} - \\ &\quad \frac{(p_1 + 2)^2 + (p_2 + 2)^2}{(p_1 + 2)(p_2 + 2)} > 0 \end{aligned}$$

Since by lemma 1 and lemma 2 of [3] $\alpha(p_1) > \alpha(p_2 - 1)$, therefore $p_1 > p_2 - 1$. Similarly, $f(p_3 + 2, p_1 + 3) > f(p_4 + 2, p_2 + 2)$ and $f(p_4 + 2, p_1 + 3) > f(p_3 + 2, p_2 + 2)$ and also $\phi(p_1 + 3, p_2 + 1) > \phi(p_1 + 2, p_2 + 2)$.

Hence this completes the proof.

3 MAXIMUM VALUES OF SDD INDEX WITH FIVE NON-PENDANT VERTICES

In this section, we determine n -vertex tricyclic graphs with five non-pendant vertices.

Theorem 3.1: Let $G \in TrG_{5,p}(p_1, p_2, p_3, p_4, p_5)$

(i) If $p_2 \geq 2$ then $SDD(TrG_{5,p}(p_1 + 1, p_2 - 1, p_3, p_4, p_5)) >$

- $SDD(G)$
- (ii) If $p_4 \geq 2$ then $SDD(TrG_{5,p}(p_1, p_2, p_3 + 1, p_4 - 1, p_5)) > SDD(G)$
- (iii) If $p_5 \geq 2$ then $SDD(TrG_{5,p}(p_1, p_2, p_3 + 1, p_4, p_5 - 1)) > SDD(G)$

Proof: (i) Assume that $G' = Q_{4,p}(p_1 + 1, p_2 - 1, p_3, p_4, p_5)$ where $p_2 \geq 2$ then

$$\begin{aligned} SDD(G') - SDD(G) &= [p_1\phi(1, p_1 + 4) - (p_1 - 1)\phi(1, p_1 + 3)] + \\ &\quad [(p_2 - 2)\phi(1, p_2 + 2) - (p_2 - 1)\phi(1, p_2 + 3)] + \\ &\quad [\phi(p_1 + 4, p_2 + 2) - \phi(p_1 + 3, p_2 + 3)] + \\ &\quad [\phi(p_1 + 4, p_3 + 1) - \phi(p_1 + 3, p_3 + 1)] + \\ &\quad [\phi(p_1 + 4, p_4 + 1) - \phi(p_1 + 3, p_4 + 1)] + \\ &\quad [\phi(p_1 + 4, p_5 + 1) - \phi(p_1 + 3, p_5 + 1)] + \\ &\quad [\phi(p_2 + 2, p_3 + 1) - \phi(p_2 + 3, p_3 + 1)] + \\ &\quad [\phi(p_2 + 2, p_5 + 1) - \phi(p_2 + 3, p_5 + 1)] + \\ &\quad [\phi(p_2 + 2, p_4 + 1) - \phi(p_2 + 3, p_4 + 1)] \\ &= \alpha(p_1) - \alpha(p_2 - 1) + f(p_3 + 1, p_1 + 4) + \\ &\quad f(p_4 + 1, p_1 + 4) + f(p_5 + 1, p_1 + 4) - \\ &\quad f(p_3 + 1, p_2 + 3) - f(p_5 + 1, p_2 + 3) - \\ &\quad f(p_4 + 1, p_2 + 3) - \frac{(p_1 + 4)^2 + (p_3 + 1)^2}{(p_1 + 4)(p_3 + 1)} - \\ &\quad \frac{(p_1 + 3)^2 + (p_3 + 1)^2}{(p_1 + 3)(p_3 + 1)} > 0 \end{aligned}$$

Since by lemma 1 and lemma 2 of [3] we get the result.

(ii) Suppose that $G' = Q_{4,p}(p_1, p_2, p_3 + 1, p_4 - 1, p_5)$ and $p_4 \geq 2$, then

$$\begin{aligned} SDD(G') - SDD(G) &= [p_3\phi(1, p_3 + 2) - (p_3 - 1)\phi(1, p_3 + 1)] + \\ &\quad [(p_4 - 2)\phi(1, p_4) - (p_4 - 1)\phi(1, p_4 + 1)] + \\ &\quad [\phi(p_1 + 3, p_3 + 2) - \phi(p_1 + 3, p_3 + 1)] + \\ &\quad [\phi(p_1 + 3, p_4) - \phi(p_1 + 3, p_4 + 1)] + \\ &\quad [\phi(p_2 + 3, p_3 + 2) - \phi(p_2 + 3, p_4 + 1)] + \\ &\quad [\phi(p_2 + 3, p_4) - \phi(p_2 + 3, p_3 + 1)] \\ &= \alpha(p_3) - \alpha(p_4 - 1) + f(p_3 + 2, p_1 + 3) - \\ &\quad f(p_4 + 1, p_1 + 3) - f(p_4 + 1, p_2 + 3) + \\ &\quad f(p_3 + 2, p_2 + 3) > 0 \end{aligned}$$

Hence the result.

Similar arguments are followed for the case (iii) to obtain the result.

4 MAXIMUM VALUES OF SDD INDEX OF TETRACYCLIC GRAPHS

Let $TG(n)$ and $TG(n, p)$, $0 \leq p \leq n - 5$ denote the set of all n -vertex tetracyclic graphs [4]. In this section we determined the maximum values of SDD index of n -vertex tetracyclic graphs.

Considering the complete graph K_4 and construct a graph M_4 by adding a vertex v_5 and connecting it to the adjacent vertices of K_4 . Since we have $d(v_1) = d(v_2) = 4$, $d(v_3) = d(v_4) = 3$ and $d(v_5) = 2$.

Let us suppose that $F_{n,p}(p_1, p_2, p_3, p_4, p_5)$ is a graph obtained from M_n by attaching $\sum_{i=1}^r p_i = n - r$ vertices.

Theorem 4.1: Let $G = F_{5,p}(p_1, p_2, p_3, p_4, p_5)$ then

- (i) $SDD(G) < SDD(F_{5,p}(p_1 + 1, p_2 - 1, p_3, p_4, p_5))$ when $p_2 \geq 2$
- (ii) $SDD(G) < SDD(F_{5,p}(p_1, p_2, p_3 + 1, p_4 - 1, p_5))$ when $p_4 \geq 2$.

Theorem 4.2: If $G = F_{6,p}(p_1, p_2, p_3, p_4, p_5, p_6)$ then we have

- (i) If $p_2 \geq 2$ then
 $SDD(G) < SDD(F_{6,p}(p_1 + 1, p_2 - 1, p_3, p_4, p_5, p_6))$
- (ii) If $p_4 \geq 2$ then
 $SDD(G) < SDD(F_{6,p}(p_1, p_2, p_3 + 1, p_4 - 1, p_5, p_6))$
- (iii) If $p_5 \geq 2$ then
 $SDD(G) < SDD(F_{6,p}(p_1, p_2, p_3 + 1, p_4, p_5 - 1, p_6))$
- (iv) If $p_6 \geq 2$ then
 $SDD(G) < SDD(F_{6,p}(p_1, p_2, p_3 + 1, p_4, p_5, p_6 - 1))$

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REFERENCES

- [1] V. Alexander, “Upper and lower bounds of symmetric division deg index”, Iranian Journal of Mathematical Chemistry vol. 5(2) (2014), pp. 91-98.
- [2] A.R. Ashra , T. Dehghan-zadeh, N. Habibi and P.E. John, “Maximum values of atom-bond connectivity index in the class of tricyclic graphs”, J. Appl. Math. Comput., vol. 50, (2016), pp. 511-527.
- [3] C.K.Gupta, V. Lokesh, Shwetha Shetty. B and Ranjini P.S, “On the Symmetric division deg index of graph”, Southeast Asian Bulletin of Mathematics, vol. 41(1), (2016), pp. 1-23.
- [4] T. Dehghan-zadeh, A.R. Ashra and N. Habibi, “Maximum values of atom-bond connectivity index in the class of tetracyclic graphs”, J. Appl. Math. Comput., vol. 46, (2014), pp. 285-303.
- [5] Damir Vukičević and Marija Gašperov, “Bond Additive Modeling 1. Adriatic Indices”, Croat. Chem. Acta, vol. 83(3) (2010), pp. 243-260.
- [6] V. Lokesh, A. Usha, P. S. Ranjini and K. M. Devendraiah, “Topological indices on model graph structure of Alveoli in human lungs”, Jang. Math. Soc., vol. 18 (4) (2015), pp. 435 - 453.
- [7] V. Lokesh, A. Usha, P. S. Ranjini and T. Deepika, “Harmonic Index of Cubic Poly-hedral Graphs Using Bridge Graphs”, App. Math. Sci., vol. 9 (2015), pp. 4245 - 4253.
- [8] P. S. Ranjini, A. Usha, V. Lokesh and T. Deepika, “Harmonic Index, Redefined Zagreb Indices of Dragon Graph with Complete Graph”, Asian J. of Math. and Comp. Research, vol. 9 (2), (2016), pp. 161 - 166.
- [9] Rundan Xing, Bo Zhou and Fengming, “On atom-bond connectivity index of connected graphs”, Discrete Appl. Math., vol. 129,(2011), pp. 1617-1630.
- [10] B. Shwetha Shetty, V. Lokesh and P. S. Ranjini,
- “On the Harmonic index of graph operations”. Transactions on Combinatorics, Vol. 4(4), (2015), pp. 5-14.
- [11] B. Shwetha Shetty, V. Lokesh, P. S. Ranjini and K.C. Das, “Computing some Topo-logical indices of smart polymer”, Digest Journal of Nanomaterials and Biostructures, vol. 7(3), (2012), pp. 1097-1102.
- [12] B. Shwetha Shetty, V. Lokesh, Bayad Abdelmejid and P. S. Ranjini, “A comparative study of topological indices and molecular weight of some carbohydrates”, J. of Indian academy of math., vol. 32, (2012), pp. 2627-2636.
- [13] R. Todeschini, V. Consonni, “Hand Book of Molecular Descriptors”, Wiley VCH, Weinheim 2000.
- [14] R. Todeschini and V. Consonni, “New local vertex invariants and molecular descriptors based on functions of the vertex degrees”, MATCH Commun. Math. Comput. Chem. Vol. 64 (2010), pp. 359 - 372.
- [15] B. Furtula, A. Graovac, D. Vukicev, “Augmented Zagreb index, J. Math. Chem. vol. 48 (2010), pp. 370 - 380.
- [16] Xing, R., Zhou, B., dong, F: “On atom-bond connectivity index of Connected graphs”, Discrete Appl. Math. Vol. 159, (2011), pp. 1617-1630.
- [17] Xing, R., Zhou, B: “Extremal trees with fixed degree sequence for atom-bond connectivity index”, Filomat vol. 26(4), (2012), pp. 683-688.